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The two-nucleon problem in EFT reformulated: Pion and nucleon masses as soft and hard scales

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We outline the modified formulation of baryon chiral effective field theory for nucleon-nucleon scattering and discuss the issue of a possible power counting violation by the nucleon mass. We also present the results for the quark mass dependence of the $^1S_0$ and $^3S_1$ scattering lengths and the deuteron binding energy at leading order.

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1. The method

A systematic quantum field theoretical approach to few-nucleon systems has been pioneered by Weinberg in Ref. [1], (see Refs. [2, 3] for recent review articles). Within this framework, the nucleon-nucleon (NN) potential is defined as a sum of two-nucleon-irreducible time ordered diagrams emerging in non-relativistic effective field theory (EFT). It is calculated as a series based on a systematic expansion in small parameters. A finite number of diagrams contribute to the effective potential at any finite order. The scattering amplitude is obtained by solving the Lippmann-Schwinger (LS) or, equivalently, Schrödinger equation.

The problem of renormalization turned out to be highly non-trivial in Weinberg's approach. To resolve this problem we have recently suggested a new framework based on the manifestly Lorentz invariant effective Lagrangian and time ordered perturbation theory [4, 5]. In this approach the leading-order amplitude is obtained by solving the integral equation

$$T_0\left(\vec{p}\,',\vec{p}\right)=V_0\left(\vec{p}\,',\vec{p}\right)-\frac{m^2}{2}\int\frac{d^3k}{(2\pi)^3}\frac{V_0\left(\vec{p}\,',\vec{k}\right)\,T_0\left(\vec{k},\vec{p}\right)}{(k^2+m^2)\left(E-\sqrt{k^2+m^2}+i\varepsilon\right)}, \tag{1.1}$$

where $E=\sqrt{p^2+m^2}$ denotes the energy of a single nucleon in the center of mass frame. Here and in what follows, we use the notation $p\equiv|\vec{p}\,|$, $k\equiv|\vec{k}\,|$. The leading order (LO) NN potential can be taken in the usual form

$$V_0\left(\vec{p}\,',\vec{p}\right)=C_S+C_T\,\vec{\sigma}_1\cdot\vec{\sigma}_2-\frac{g_A^2}{4F^2}\,\vec{\tau}_1\cdot\vec{\tau}_2\,\frac{\vec{\sigma}_1\cdot(\vec{p}\,'-\vec{p})\,\vec{\sigma}_2\cdot(\vec{p}\,'-\vec{p})}{(\vec{p}\,'-\vec{p})^2+M_\pi^2}, \tag{1.2}$$

where the standard notation is employed, see [4] for more details. Notice that equation (1.1) was first obtained in Ref. [6]. Its iterations for the potential (1.2) generate only logarithmic divergences which can be absorbed into redefinition of the couplings C_S and C_T, i.e. it is perturbatively renormalizable. Partial wave projected equations corresponding to Eq. (1.1) have unique solutions except for the $^3P_0$ channel. We solved the problem of non-uniqueness of the solution in this partial wave analogously to Ref. [7] by including a counter term of the form $C(\Lambda)p'p/\Lambda^2$, with $C(\Lambda)$ being a cutoff dependent constant, in the leading-order potential [4].

2. Iterations of one-pion exchange and the role of the nucleon mass

Comparing Eq. (1.1) with the Lippmann-Schwinger equation of the non-relativistic heavy baryon chiral perturbation theory [1] one might come to the (wrong) conclusion that the nucleon mass plays the role of the cutoff in Eq. (1.1) and, being the hard scale, violates the power counting. We now analyze the nucleon-mass dependence of renormalized loop diagrams by considering the first two iterations of the one-pion exchange (OPE) potential in order to demonstrate explicitly that the above naive interpretation is misleading.

We first consider the dimensionally regularized one-loop tensor integral corresponding to a single iteration of the OPE potential

$$I_1=-\frac{m^2}{2}\int\frac{d^nk\,\left(p'_a-k_a\right)\left(p'_b-k_b\right)\left(p_i-k_i\right)\left(p_j-k_j\right)}{(k^2+m^2)\left(E-\sqrt{k^2+m^2}\right)\left[(p'-k)^2+M^2\right]\left[(p-k)^2+M_\pi^2\right]}, \tag{2.1}$$

where we drop, for the sake of simplicity, the $+i\varepsilon$ prescription for two-nucleon propagators. Its expansion in inverse powers of the nucleon mass can be easily obtained by applying the method of dimensional counting [8] and has the form

$$I_1 = m^2 m^{n-3} \int \frac{d^n q \; q_a q_b q_i q_j}{2q^4 \left[q^2+1\right] \left[\sqrt{q^2+1}-1\right]} + \ldots, \tag{2.2}$$

where q is a (re-scaled) dimensionless integration variable and the ellipses refer to terms of higher orders in the $1/m$-expansion. The first term in Eq. (2.2) is momentum and pion-mass independent and is cancelled by a counter-term associated with the coupling constants of the LO contact interactions. Higher-order terms in Eq. (2.2) have either the same linear dependence on the nucleon mass (for spacetime dimension $n = 3$) as in the heavy-baryon expression, or are suppressed by additional inverse powers of m. Thus, the nucleon mass does not violate the power counting in our new approach at one loop level.

We now turn to the two-loop tensor integral emerging from the second iteration of the OPE potential

$$\begin{aligned} I_2 = \frac{m^4}{4} \int & \frac{d^n k_1 d^n k_2 \left(p'_a - k_{1a}\right)\left(p'_b - k_{1b}\right)\left(k_{1i}-k_{2i}\right)\left(k_{1j}-k_{2j}\right)\left(k_{2\mu}-p_\mu\right)\left(k_{2\nu}-p_\nu\right)}{\left(k_1^2+m^2\right)\left(k_2^2+m^2\right)\left(E-\sqrt{k_1^2+m^2}\right)\left(E-\sqrt{k_2^2+m^2}\right)} \\ \times & \frac{1}{\left[(p'-k_1)^2+M_\pi^2\right]\left[(k_1-k_2)^2+M_\pi^2\right]\left[(p-k_2)^2+M_\pi^2\right]}. \end{aligned} \tag{2.3}$$

Its expansion in inverse powers of the nucleon mass has the form [8]

$$\begin{aligned} I_2 = & \frac{m^4 m^{2n-6}}{4} \int \frac{d^n q_1 d^n q_2 \; q_{1i} q_{1j} q_{2\mu} q_{2\nu} \left(q_{1a}-q_{2a}\right)\left(q_{1b}-q_{2b}\right)}{q_1^2\left(1+q_1^2\right)\left[\sqrt{1+q_1^2}-1\right] q_2^2 \left(1+q_2^2\right)\left[\sqrt{1+q_2^2}-1\right]\left(q_1-q_2\right)^2} \\ & + \frac{m^3 m^{n-3}}{2} \int \frac{d^n q_1 d^n k_2 \; q_{1a} q_{1b} q_{1i} q_{1j} \left(k_{2\mu}-p_\mu\right)\left(k_{2\nu}-p_\nu\right)}{\left(k_2^2-p^2\right) q_1^4 \left(1+q_1^2\right)\left[\sqrt{1+q_1^2}-1\right]\left[(p-k_2)^2+M_\pi^2\right]} \\ & + \frac{m^3 m^{n-3}}{2} \int \frac{d^n q_2 d^n k_1 \; q_{2a} q_{2b} q_{2\mu} q_{2\nu} \left(k_{1i}-p'_i\right)\left(k_{1j}-p'_j\right)}{\left(k_1^2-p^2\right) q_2^4 \left(1+q_2^2\right)\left[\sqrt{1+q_2^2}-1\right]\left[(p-k_1)^2+M_\pi^2\right]} + \ldots. \end{aligned} \tag{2.4}$$

Renormalization of two-loop diagrams requires the addition of one-loop diagrams generated by a single iteration of the one-loop counter terms, see Eq. (2.2), in order to subtract the sub-divergences (and finite pieces) of one-loop sub-diagrams. Correspondingly, we need to add to the integral I_2 two counter-term integrals:

$$\begin{aligned} I_{2ct} = \frac{m^4 m^{n-3}}{4} \Bigg\{ & \int \frac{d^n q d^n k_1 \; q_a q_b q_\mu q_\nu \left(p'_i - k_{1i}\right)\left(p'_j - k_{1j}\right)}{\left(k_1^2+m^2\right)\left(\sqrt{p^2+m^2}-\sqrt{k_1^2+m^2}\right) q^4 \left(1+q^2\right)\left[\sqrt{1+q^2}-1\right]\left[(p'-k_1)^2+M_\pi^2\right]} \\ + & \int \frac{d^n q d^n k_2 \; q_a q_b q_i q_j \left(p_\mu - k_{2\mu}\right)\left(p_\nu - k_{2\nu}\right)}{\left(k_2^2+m^2\right)\left(\sqrt{p^2+m^2}-\sqrt{k_2^2+m^2}\right) q^4 \left(1+q^2\right)\left[\sqrt{1+q^2}-1\right]\left[(p-k_2)^2+M_\pi^2\right]} \Bigg\}. \end{aligned} \tag{2.5}$$

The expansion of these counter-term integrals in inverse powers of the nucleon mass has the form

$$
\begin{aligned}
I_{2ct} = & -\frac{m^4 m^{2n-6}}{4}\left\{\int \frac{d^n q_2 d^n q \; q_{2\mu} q_{2\nu} q_i q_j q_a q_b}{q_2^2\left(1+q_2^2\right)\left(\sqrt{1+q_2^2}-1\right) q^4\left(1+q^2\right)\left(\sqrt{1+q^2}-1\right)}\right. \\
& \left. + \int \frac{d^n q_1 d^n q \; q_{1i} q_{1j} q_\mu q_\nu q_a q_b}{q_1^2\left(1+q_1^2\right)\left(\sqrt{1+q_1^2}-1\right) q^4\left(1+q^2\right)\left(\sqrt{1+q^2}-1\right)}\right\} \\
& - \frac{m^3 m^{n-3}}{2}\left\{\int \frac{d^n q d^n k_2 \; q_a q_b q_i q_j \left(k_{2\mu}-p_\mu\right)\left(k_{2\nu}-p_\nu\right)}{\left(k_2^2-p^2\right) q^4\left(1+q^2\right)\left(\sqrt{1+q^2}-1\right)\left[(p-k_2)^2+M_\pi^2\right]}\right. \\
& \left. + \int \frac{d^n q_2 d^n k_1 \; q_a q_b q_\mu q_\nu \left(k_{1i}-p_i'\right)\left(k_{1j}-p_j'\right)}{\left(k_1^2-p^2\right) q^4\left(1+q^2\right)\left(\sqrt{1+q^2}-1\right)\left[(p-k_1)^2+M_\pi^2\right]}\right\} + \ldots . \qquad (2.6)
\end{aligned}
$$

Adding the expressions in Eqs. (2.4) and (2.6) together we observe that all terms proportional to $m^3 m^{n-3}$ cancel exactly. The remaining terms proportional to $m^4 m^{2n-6}$ are momentum and pion-mass independent. They are subtracted by the two-loop counter-terms generated by coupling constants of the LO contact potential. The remaining terms have either the same quadratic dependence on the nucleon mass as in the heavy-baryon approach or are suppressed by additional inverse powers of m. Analogously, it can be shown for any number of iterations that the hard dependence on the nucleon mass is removed by renormalization yielding the amplitude which obeys the standard power counting of the heavy-baryon approach. The naive interpretation of the nucleon mass playing the role of the cutoff in our new formulation is misleading because it is based on the comparison with the heavy-baryon result ignoring the crucial fact that the heavy-baryon expansion does not commute with the expansion in inverse powers of the cutoff parameter.

3. Pion-mass dependence at leading order

As our new framework is perturbatively renormalizable and we do not attempt to integrate out the momentum scale $\sim \sqrt{m M_\pi}$, see Ref. [10], there is no implicit quark- (pion-) mass dependence of coupling constants associated with contact interactions. Therefore, we can straightforwardly calculate the quark-mass dependence of two-nucleon observables order-by-order in the chiral expansion. Below we present an exploratory investigation of chiral extrapolations of the deuteron binding energy and the scattering lengths in the $^1S_0$ and $^3S_1$ partial waves at LO.

There is one free parameter in each of the S-waves at this order. These parameters are given by linear combinations of the low-energy constants C_S and C_T in Eq. (1.2) and are fitted to phase shifts of the Nijmegen partial wave analysis at low energy, see Ref. [4] for more detail. The quark- or, equivalently, pion-mass dependence of the NN amplitude at LO is entirely driven by the explicit pion-mass dependence of the OPE potential. Figure 1 shows the resulting quark-mass dependence of the deuteron binding energy together with the recent results of Ref. [11], see also Refs. [12, 13, 14, 15] for some earlier EFT calculations along these lines. Given the theoretical accuracy of our LO analysis and of the calculation of Ref. [11] which relies on the resonance saturation hypothesis for contact interactions, the agreement can be regarded as excellent. We have also calculated the

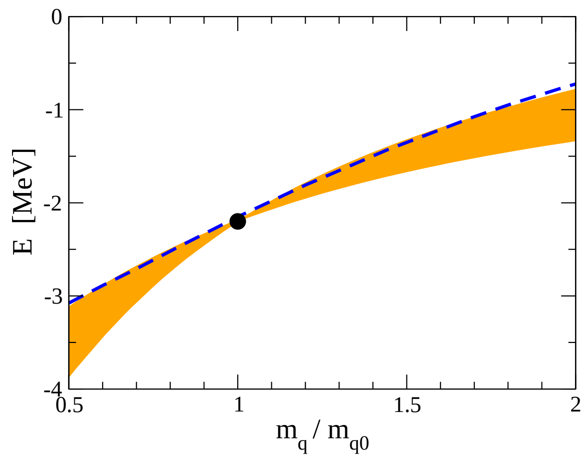

Figure 1: Quark-mass dependence of the deuteron binding energy. The dashed line corresponds to the LO of the modified Weinberg approach and the light-shaded band to N^{2}LO result from Ref. [11]. The band corresponds to the cutoff variation. The solid dot shows the deuteron binding energy at the physical value of the quark mass.

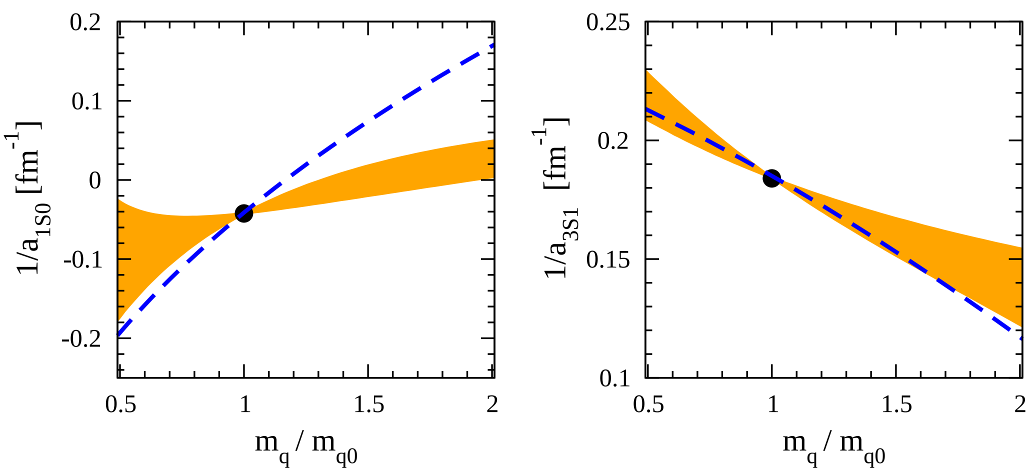

Figure 2: Quark-mass dependence of the inverse S-wave scattering lengths of $^1S_0$ and $^3S_1$ partial waves. The dashed lines correspond to the LO of the modified Weinberg approach and the light-shaded bands to N^{2}LO results from Ref. [11]. The bands correspond to the cutoff variation. The solid dots show the inverse scattering lengths at the physical value of the quark mass.

quark-mass dependence of the inverse scattering lengths which is shown in Fig. 2. For the singlet scattering length we observe the same qualitative behavior as found in Ref. [11]. Somewhat larger deviations from the N^{2}LO analysis of that work are not surprising in view of the fact that the OPE potential plays a fairly minor role in this channel.

4. Summary

In this conference contribution we outlined the modified formulation of baryon chiral perturbation theory for nucleon-nucleon interaction [4] and analyzed iterations of the leading order

one-pion exchange potential. Naively, it might appear that the nucleon mass plays the role of an ultraviolet cutoff in our approach. We have shown that this interpretation is misleading as it ignores the fact that the heavy-baryon expansion does not commute with the expansion in inverse powers of the cutoff parameter. For one- and two-loop diagrams corresponding to iterations of the OPE potential, we explicitly demonstrated that renormalization indeed removes all nucleon-mass dependence which violates the power counting.

As an application of our approach, we explored the quark-mass dependence of the deuteron binding energy and the S-wave scattering lengths at LO in the EFT expansion. The obtained results are in a good agreement with the recent calculation of Ref. [11].

Acknowledgments

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